XVII Congreso Galego de Estatística e Investigación de Operacións
Ourense, 23 - 25 de outubro de 2025

TUGLab, a laboratory for TU games

Álvaro de Prado Saborido[1], Alejandro Bernárdez Ferradás[1], Miguel Ángel Mirás Calvo[2], Iago Núñez Lugilde[3], Carmen Quinteiro Sandomingo[2], Estela Sánchez Rodríguez[1]

[1] SiDOR. Dep. de Estatística e Investigación Operativa, Universidade de Vigo. CITMAga.
[2] Dep. de Matemáticas, Universidade de Vigo.
[3] MODES. Dep. de Matemáticas, Universidade da Coruña.

ABSTRACT

Cooperative game theory models decision-making situations in which a group of agents, called players, may achieve certain benefits by cooperating to reach an optimal outcome. It has great potential in different fields, since it offers a framework to analyze and solve problems in which cooperation is essential to achieve a common goal.

The TUGLab (Transferable Utility Games Laboratory) **R** package [3] contains a set of functions that may serve as a helpful complement to the books and other materials used in courses on cooperative game theory, and also as a practical tool for researchers working in this field. The TUGLab project was born in 2006 with the purpose of highlighting the geometrical aspects of cooperative games with 3 and 4 players. TUGlabWeb [5], an online platform on which basic functions are implemented, is being used all over the world as a resource in degree, master's and doctoral programs.

The current TUGLab package is an extension of previous works that enables users to work with games more generally, computational restrictions aside. Users can check game properties, compute well-known games and obtain several set-valued and single-valued solutions, such as the core, the Shapley value, the nucleolus or the core-center. Some TUGLab functions aim to illustrate the flexibility of the Shapley value, which can be adapted to fit various cooperative game settings, including weighted players and coalitions, a priori unions, or restricted communication structures. In keeping with the original philosophy of previous versions, special emphasis is placed on the graphical representation of the solution concepts for 3 and 4 players.

There are other **R** packages centered on cooperative games in general, such as GameTheory [2], CoopGame [7] or TUvalues [4]. Some others are more specific and deal with particular types of problems and their associated games. ClaimsProblems [6] is dedicated exclusively to problems of allocation of scarce resource, such as bankruptcy situations, tax distribution, or CO_2 emissions distribution. AirportProblems [1] is devoted to cost allocation problems in which agents share the cost of a facility or service.

Keywords: cooperative games, sharing rules, set solutions, graphical visualization.

ACKNOWLEDGEMENTS

This work was supported by project PID2021-124030NB-C33, which is funded by MCIN/AEI/10.13039/501100011033/ and "ERDF A way of making Europe"/EU, and by Grupo de Referencia Competitiva de la Xunta de Galicia - Programa de Consolidación - GRC-ED431C 2024/26.

REFERENCES

[1] Bernárdez Ferradás, A., Sánchez Rodríguez, E., Mirás Calvo, M. Á., and Quinteiro Sandomingo, C. (2025). *AirportProblems: Analysis of Cost Allocation for Airport Problems*, **R** package version 0.1.0.
`https://CRAN.R-project.org/package=AirportProblems`

[2] Cano-Berlanga, S., Gimenez-Gomez, J. M., and Vilella, C. (2017). Enjoying cooperative games: The **R** package GameTheory. *Applied Mathematics and Computation* 305, 381-393.

[3] de Prado Saborido, Á., Mirás Calvo, M. Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2025). *TUGLab: A Laboratory for TU Games*, **R** package version 0.0.1.
`https://CRAN.R-project.org/package=TUGLab`

[4] Guillén, M. D., Gonçalves, J. (2025). *TUvalues: Tools for Calculating Allocations in Game Theory using Exact and Approximated Methods*, **R** package version 1.0.0.
`https://CRAN.R-project.org/package=TUvalues`.

[5] Mirás Calvo, M. Á. and Sánchez Rodríguez, E. (2010). Herramientas informáticas de cálculo y representación gráfica para juegos TU. *La Gaceta de la RSME* 13 (1), 89–108.
`http://tuglabweb.uvigo.es/TUGlabWEB2/index.php`

[6] Sánchez Rodríguez, E., Mirás Calvo, M. Á., Núñez Lugilde, I., and Quinteiro Sandomingo, C. (2025). *ClaimsProblems: Analysis of Conflicting Claims*, **R** package version 1.0.0.
`https://cran.r-project.org/package=ClaimsProblems`

[7] Staudacher, J. and Anwander, J. (2019). Using the **R** package CoopGame for the analysis, solution and visualization of cooperative games with transferable utility; **R** Vignette.